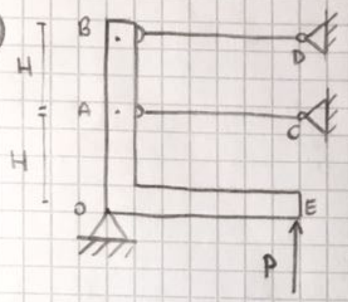
ESERCITAZIONE 1 - 28/10/2016

① 

CALCOLA LO STATO DI TENSIONE SU BD e AC?

BD e AC soggetti a $\Delta T_{BD} = \Delta T_{AC}$

In P.D. posso sommare gli effetti:

$\cdot\ \delta_1\big|_{AC} = \alpha \Delta T L_1 + \frac{P_1 L_1}{EA_1}$

$\cdot\ \delta_2\big|_{BD} = \alpha \Delta T_2 L_2 + \frac{P_1 L_2}{EA_2}$

$\cdot\ \delta_2 = 2 \cdot \delta_1$ (congruenza)

DATI: $A_1 = A_2$
$L_1 = L_2$
$\Delta T_{BD} = \Delta T_{AC}$
(le barre si riscaldano)

$$\rightarrow \left(\alpha \Delta T L_1 + \frac{P_1 L_1}{EA_1}\right)2 = \alpha \Delta T L_2 + \frac{P_2 L_2}{EA_2}$$

$$\rightarrow P_2 = \alpha \Delta T \cdot LEA + 2P_1$$

· (equilibrio): $\sum M_o = 0 \quad \rightarrow PL = P_1 H + 2P_2 H$

$$\rightarrow PL = P_1 H + 2\alpha \Delta T EAH + 4P_1 H$$

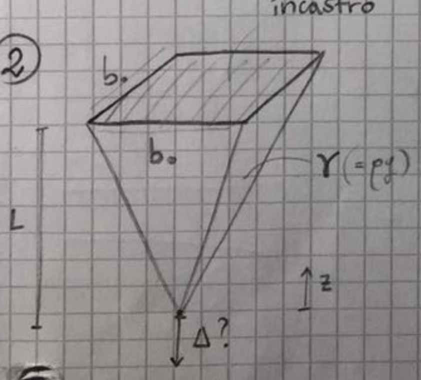
$$\rightarrow P_1 = \frac{PL}{5H} - \frac{2\alpha \Delta T EA}{5}$$

→ sostituisco e trovo P_2 → (...)

DATI NUMERICI: $L = 1m$, $A = 1\,cm^2$, $P = 5000N$, $E = 2{,}1 \times 10^{11}\,Pa$

$\Delta T = 100°$, $\alpha = 10^{-5}\,(°^{-1})$, $H = 0{,}5\,m$

→ $P_1 = -6400\,N$, $\sigma_1 = -64\,MPa$
$P_2 = +8200\,N$, $\sigma_2 = 82\,MPa$

② 

CALCOLA L'ALLUNGAMENTO DOVUTO AL SUO PESO

$$P(z) = \frac{1}{3} \cdot A(z) \cdot z\gamma$$

$$\underline{\delta} = \int_0^L \varepsilon\, dz = \int_0^L \frac{\sigma}{E}\, dz = \int_0^L \frac{P}{A(z) \cdot E}\, dz =$$

$$= \int_0^L \frac{1}{3} \frac{A \cdot z \cdot \gamma}{AE}\, dz =$$

[OSS.: L'area è $A = b(z)^2$ ma tanto si SEMPLIFICA!]

$$= \frac{\gamma}{3E} \int_0^L z\, dz = \underline{\frac{\gamma L^2}{6E}}$$

(oss. non dipende da b_o!)

→ affinché le due formule di ϑ siano uguali nei due casi:

$$\rightarrow J' = \frac{4 A_m^2}{\oint_{l.m.} \frac{dc}{s}}$$

$$\ln\left[\frac{5/4\,(a-b)}{3/4\,(a-b)}\right]$$

$$\rightarrow s(c) = s_{min} + \frac{\Delta s \cdot \vartheta}{\pi}$$

$$\rightarrow \oint \frac{dc}{s} = 2\int_0^{\pi} \frac{\left(\frac{a+b}{2}\right) d\vartheta}{s_{min} + \frac{\Delta s \cdot \vartheta}{\pi}} = (a+b)\int_0^{\pi} \frac{d\vartheta}{s_{min} + \frac{\Delta s \cdot \vartheta}{\pi}} = \frac{\pi(a+b)}{\Delta s} \cdot \left[\ln\left(s_{min} + \frac{\Delta s\,\vartheta}{\pi}\right)\right]_0^{\pi} =$$

$$\rightarrow \vartheta' = \frac{T}{G \cdot 4A_m^2} \oint \frac{dc}{s} = \frac{T}{G} \cdot \frac{4}{\pi^2 (a+b)^4} \cdot \pi \, \frac{2(a+b)}{a-b} \cdot \ln\left(\frac{5}{3}\right) =$$

$$= \frac{T}{G} \cdot \frac{8}{\pi (a+b)^2 (a^2-b^2)} \cdot \ln\left(\frac{5}{3}\right)$$

$$\rightarrow \frac{\vartheta'}{\vartheta} = \frac{4(a^2+b^2)}{(a+b)^2} \cdot \ln\frac{5}{3} = 1{,}035$$

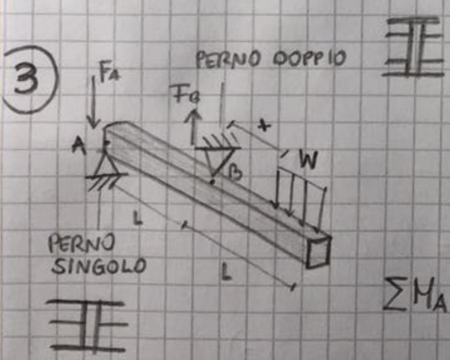
→ l'effetto sulla rigidezza è inferiore

$(*)$: $J_{pol} = \pi \frac{(a^4 - b^4)}{2} = 9{,}274 \cdot 10^7 \text{ mm}^4$

$J_{Bredt} = 2\pi\left(\frac{a+b}{2}\right)(a-b)\frac{(a+b)^2}{4} = \left(2\pi\bar{r} \cdot s \cdot \bar{r}^2\right) = 9{,}161 \cdot 10^7 \text{ mm}^4$

→ valori simili

3



DATI: $\tau_{amm} = 150$ MPa per i PERNI, $d_p = 8$ mm

$W = 8$ kN/m, $L = 2$ m

INC: x t.c. entrano in crisi i perni

X_A (crisi A)

X_B (crisi B)

$$\Sigma M_A = 0 \rightarrow F_B L - W \cdot (L-x) \cdot \underbrace{\left(L + x + \left(\frac{L-x}{2}\right)\right)}_{(3L + x)/2} = 0$$

$$\text{(I)} \rightarrow 2F_B - 48 + 16x + 4x^2 = 0 \quad (2° \text{ grado})$$

$$\Sigma M_B = 0 \rightarrow F_A \cdot L - W(L-x)\underbrace{\left(x + \frac{L-x}{2}\right)}_{(L+x)/2} = 0$$

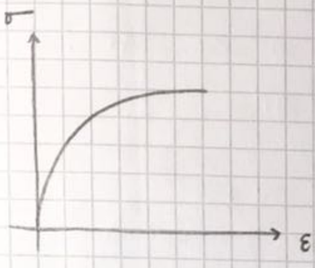
$$\text{(II)} \rightarrow 2F_A - 16 + 4x^2 = 0$$

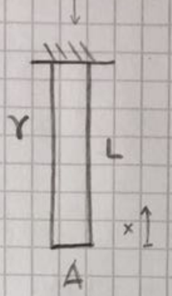
(II)

$$\rightarrow \tau = \frac{F_A}{A} \longleftrightarrow 150 \cdot 10^6 = \frac{F_A}{\frac{\pi}{4}(0{,}008)^2} \rightarrow F_A = 7539 \text{ N}$$

→ sostituisco F_A nella (II) → $\underline{X_A = 0{,}480 \text{ m}}$

③ PROVA DI TRAZ. SU UN MATERIALE : $\sigma = C \cdot \sqrt{\varepsilon}$ → TROVARE δ



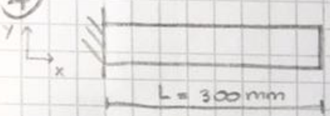


$$\sigma^2 = \frac{P(x)}{A} \qquad \varepsilon = \frac{d\delta}{dx}$$

$$\frac{P(x)^2}{A^2} = C^2 \frac{d\delta}{dx} \qquad (\text{EQ. DIFF.})$$

→ separo le variabili : $\underline{\delta} = \int_0^L d\delta = \frac{1}{A^2C^2}\int_0^L P^2(x)\,dx = \frac{1}{A^2C^2}\int_0^L (\gamma A x)^2 dx = \underline{\frac{\gamma}{C^2}\cdot\frac{L^3}{3}}$

④



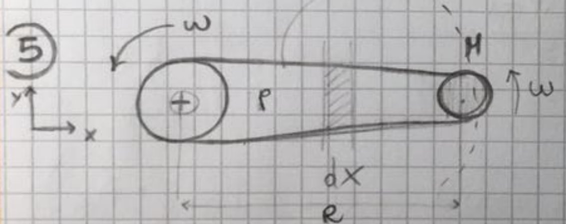
- TRAVE SOGGETTA A TAGLIO
- LA DEFORMAZIONE E' NOTA : $\underline{\gamma_{xy}(x) = 0{,}02 \cdot x}$

$\Delta y_{max} = ?$

$$\frac{dy}{dx} = tg\,\gamma_{xy} = tg(0{,}02x) \qquad \left[\int tg\,x = \int \frac{\sin x}{\cos x}dx = \int \frac{f'(x)}{f(x)}dx = \ln\left(\frac{1}{f(x)}\right)\right]$$

$$\Delta y = \int_0^y dy = \int_0^L tg(0{,}02x)\,dx = -50 \cdot \ln(\cos(0{,}02x))\Big|_0^{300} = 2{,}03 \text{ mm}$$

⑤



ELEMENTO PIENO : LA MASSA M RUOTA ATTORNO A ⊕

→ AGISCE FORZA CENTRIFUGA
→ FISSO UNA σ_0, ω

$A(x) = ?$ AFFINCHE' SI ABBIA UNIFORME RESISTENZA

$$-\sigma_0 A + \sigma_0(A + dA) + \omega^2 x \rho A\,dx = 0$$

$$\to \frac{dA}{A} = -\frac{\rho\omega^2 x\,dx}{\sigma_0} \quad \to \quad \underline{A = C \cdot e^{-\frac{\rho\omega^2 x^2}{2\sigma_0}}}$$

$$\left[\begin{array}{l} \underline{\text{impongo}}\ \underline{\text{B.C.}} : \quad M\omega^2 R = A_f \cdot \sigma_0 \\ \to \dfrac{M\omega^2 R}{\sigma_0} = A_f = C \cdot e^{-\left(\frac{\rho\omega^2 R^2}{2\sigma_0}\right)} \quad \to \quad C = \ldots \end{array}\right]$$

$$\to A(x) = \frac{M\omega^2 R}{\sigma_0} \cdot e^{\frac{\rho\omega^2}{2\sigma_0}(R^2 - x^2)} \to$$

- andamento -

(o 15000 ?)

DATI : $M = 10$ Kg $\quad n = 7500$ g/min
$R = 1$ m $\quad \sigma_0 = 500$ MPa
$\rho = 2700 \frac{kg}{m^3}$ $\quad \omega = \frac{2\pi n}{60} = 523{,}6 \frac{rad}{s}$

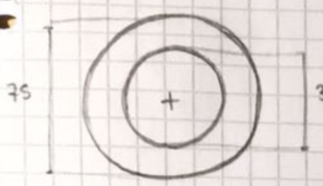
$$A_f = \frac{M\omega^2 R}{\sigma_0} = 0{,}0123 \text{ m}^2$$

→ $A_0\big|_{x=0} = 0{,}0652$ m²

→ $r_0 = 144{,}1$ mm, $r_f = 62{,}66$ mm

ESERCITAZIONE 2 - 21-11-2016

① ALBERO CAVO



DATI: $\vartheta = 2°$ ($34{,}91 \cdot 10^{-3}$ rad), $L = 4m$, $n = 120 \frac{g}{min}$, $G = 77$ GPa

INC: POTENZA TRASMESSA W?

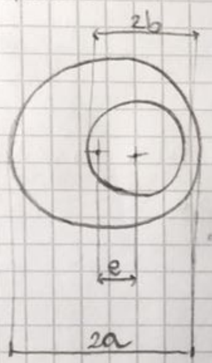
$r_1 = d_1/2 = 0{,}015\,m \qquad r_2 = d_2/2 = 0{,}0375\,m$

(PIENO - VUOTO): $J = \frac{\pi}{2}(r_1^4 - r_2^2) = 3{,}027 \cdot 10^{-6}\,m^4$

$$\vartheta = \frac{TL}{GJ} \rightarrow T = \frac{\vartheta \cdot GJ}{L} = 2{,}034 \cdot 10^3\ N \cdot m$$

$$W = T \cdot \omega = T \cdot \frac{2\pi n}{60} = 25'600\ W$$

② ALBERO CON FORO ECCENTRICO RISPETTO AL DIAMETRO ESTERNO:



DATI: $a = 100$ mm $\quad b = 80$ mm $\quad e = \frac{1}{4}(a-b)$

INC: come cambiano TENSIONI, DEF. E RIGIDEZZA dell'albero?

OSS:

Nei casi NON ASSIALSIMMETRICI non vale $\vartheta = \frac{TL}{G(J_p)}$

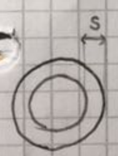
→ considero SEZIONE CHIUSA → uso BREDT

→ faccio il calcolo < senza e / con e

• TENS-DEF: τ ammissibile

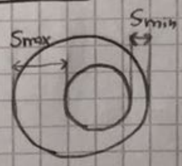
senza e:

$$\tau_0 = \frac{T}{2 \cdot s \cdot A_m} = \frac{T}{2(a-b) \cdot \left[\pi\left(\frac{a+b}{2}\right)^2\right]} \rightarrow T = 2\pi(a-b)\left(\frac{a+b}{2}\right)^2 \cdot \tau_0$$



con e:

$$s_{min} = a - b - e = a - b - \frac{1}{4}(a-b) = \frac{3}{4}(a-b)$$



→ (stessa espressione con T') $\xrightarrow{\dots}$ $T' = 2\pi\left[\frac{3}{4}(a-b)\right]\left(\frac{a+b}{2}\right)^2 \cdot \tau_0$

$$\rightarrow \frac{T}{T'} = \frac{3}{4} \quad \text{(A PARITA' DI } \tau \text{ AMMISSIBILE)}$$

(*)

• RIGIDEZZA:

- senza e: $\vartheta = \frac{T}{G \cdot J_0} \stackrel{(*)}{\cong} \frac{T}{G \cdot J_{pol}} = \frac{T}{G}\frac{32}{\pi(a^4 - b^4)}$

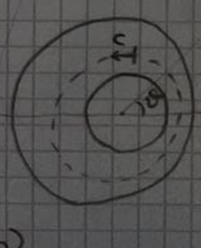
- con e: $\vartheta' = \frac{T}{4G A_m^2} \oint \frac{dc}{s} \xrightarrow{hp}$

devo risolvere ←

Area media

- non dipende da e →

→ $A_m = \pi\left(\frac{a+b}{2}\right)^2$



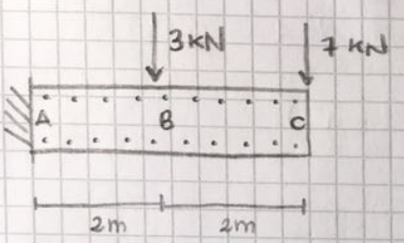
$S_{MIN} = \frac{3}{4}(a-b)$

$S_{MAX} = \frac{5}{4}(a-b) \rightarrow \Delta s = \frac{1}{2}(a-b)$

(I) $150 \cdot 10^6 = \dfrac{F_B/2}{\frac{\pi}{4}(0{,}008)^2}$ (TAGLIO DOPPIO) $\rightarrow F_B = 15\,079$ N

$\rightarrow$ sostituisco in (II) $\rightarrow$ $\underline{X_B = 0{,}909 \text{ m}}$

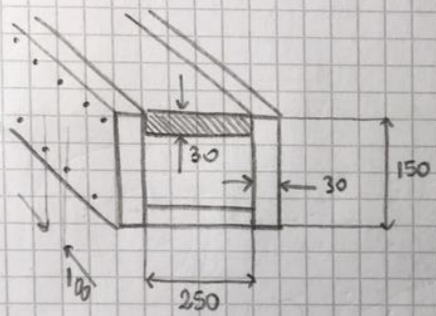
4



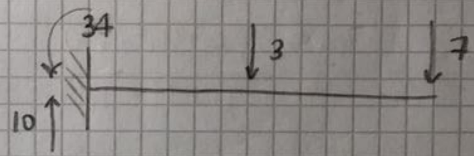
DATI: $d_{chiodi} = 5$ mm

INC: flusso di taglio

OSS: i CHIODI sono sollecitati a TAGLIO



mm



(AB): $V_{AB} = 10$ kN

(pieno-vuoto) $I = \frac{1}{12}(0{,}310)(0{,}150)^3 - \frac{1}{12}(0{,}250)(0{,}09)^3 =$

$= 72 \cdot 10^{-6}$ m^4

[OSS: faccio Q della sezione //// o in alternativa di tutto il resto]

(mom. stat.) $Q = (0{,}250 \cdot 0{,}03) \cdot 0{,}06 = 0{,}450 \cdot 10^{-3}$ m^3

$\rightarrow q = \dfrac{VQ}{I} = 62{,}5$ kN/m (FLUSSO DI TAGLIO)

calcolo F che agisce su un chiodo:

- 100 mm ZONA D'INFL.
- divido x 2 perché ho 1 CHIODO a DX e 1 a SX

$F = \dfrac{qs}{2} = \dfrac{62\,500 \cdot 0{,}100}{2} = 3125$ N

$\tau_{ch} = \dfrac{F}{A} = \dfrac{3125}{\frac{\pi}{4}(0{,}005)^2} = 159$ MPa

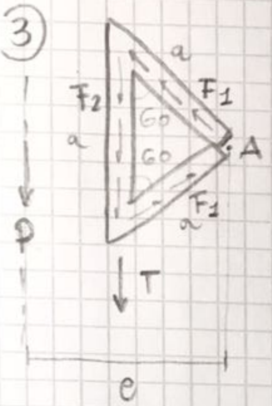
OSS: Se avessi inchiodato così:

I CHIODI NON RISENTONO DELLA COMPRESSIONE SULL'ANIMA, mentre nell'altra config. risentono della flessione.

→ $q_{max} = \frac{1}{2}\frac{VQ_{max}}{I} = \frac{1}{2}\frac{150\cdot 21{,}3\cdot 10^{-6}}{0{,}9819\cdot 10^{-6}} = 1{,}63$ kN/m

$\left(\tau = \frac{q}{s}\right)$

3)



— CALCOLO IL CENTRO DI TAGLIO —

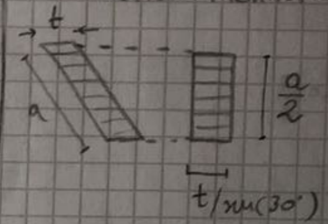
scelgo A (due risultanti avranno braccio nullo)

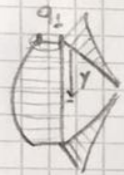
$$P\cdot e = F_2\cdot\frac{\sqrt{3}}{2}a$$

CONSIDERO I 2 TRATTI INCLINATI COME UN'UNICA BARRA VERTICALE $\frac{a}{2}+\frac{a}{2}$

$$I = \frac{1}{12}ta^3 + \frac{1}{12}\left(\frac{t}{\sin(30)}\right)a^3 = \frac{1}{4}ta^3$$

OSS: I delle sezioni inclinate



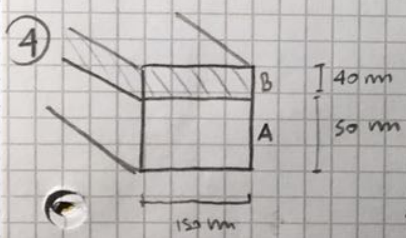


$$q_1 = \frac{P\cdot Q_1}{J} = \frac{P\cdot at\cdot a/4}{\frac{1}{4}\cdot t\cdot a^3} = \frac{P}{a}$$

$$F_2 = q_1 a + 2V\frac{\int_0^{a/2} yt\cdot\frac{a-y}{2}\,dy}{\frac{1}{4}\cdot t\cdot a^3} = P + \frac{V\left[\frac{5}{48}ta^3\right]}{\frac{1}{8}ta^3} = \frac{11}{6}P$$

$$\rightarrow e = \frac{11}{12}\cdot\sqrt{3}\cdot a$$

4)

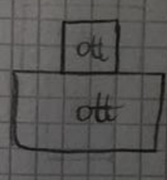


DATI: A → Al 6061-T6 → $\sigma_{amm}^{Al} = 128$ MPa

B → ott C83400 → $\sigma_{amm}^{ott} = 35$ MPa

INC: MAX mom. ELASTICO

$$n = \frac{E_{Al}}{E_{ott}} = \frac{68{,}9\cdot 10^9}{101\cdot 10^9} = 0{,}68218, \quad b_{ott} = \frac{b_{Al}}{n} = 0{,}22 \text{ mm}$$



$\bar{A}_i$: area che tiene conto del restringimento / allargamento sezione dovuto al bimateriale (vd. con [illegible])

BARICENTRO: $\bar{y} = \frac{\sum \bar{y}_i\cdot\bar{A}_i}{\sum\bar{A}_i}$

$$\bar{y} = \frac{0{,}07\,(0{,}1023\cdot 0{,}05) + 0{,}025\,(0{,}15\cdot 0{,}04)}{0{,}1023\cdot 0{,}04 + 0{,}15\cdot 0{,}05} =$$

$$I = I_{Al} + I_{ott} = 7{,}458\cdot 10^{-6}\ m^4$$

$$\sigma\big|_{ott} = \frac{Mc}{I} \leftrightarrow 35\cdot 10^6 = \frac{M\,(0{,}09 - 0{,}0493)}{7{,}458\cdot 10^{-6}} \rightarrow M = 6{,}41 \text{ kN/m}$$

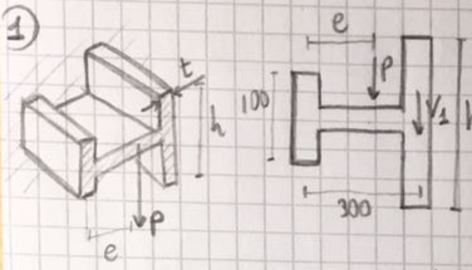
$$\big|_{Al} = n\frac{Mc}{I} \leftrightarrow M = 28{,}4 \text{ kN/m}$$

Alternativa: $\sigma_{amm\ ott} = \frac{M\cdot E_{ott}\cdot y_{ott}}{(E_{ott}\cdot I_{ott} + E_{Al}\cdot I_{Al})}$

(ESERCIZIO DA RIVEDERE!)

ESERCITAZIONE 3

(1)



DATI: $e = 250\,mm$ $\quad t = 10\,mm$

INC. $\underline{\underline{h}}$ t.c. LA TRAVE SI INFLETTA SENZA TORCERSI

Calcolo il M_{tors} sulla FLANGIA x capire come si distribuiscono le forze

$$P \cdot 0{,}250 = V_1 \cdot 0{,}300 \rightarrow V_1 = 0{,}833 \cdot P$$

trasc.

$$I = \frac{1}{12} \cdot t \cdot (0{,}1)^3 + \cancel{\frac{1}{12}(0{,}300 \cdot t^3)} + \frac{1}{12} \cdot t \cdot h^3 = \frac{t}{12}\left(10^{-3} + h^3\right)$$

$$y' = y + \frac{h/2 - y}{2} = \frac{1}{2}(y + 0{,}5 \cdot h)$$

$$Q = y' \cdot A = \frac{1}{2}(y + 0{,}5h)(0{,}5h - y) \cdot t = \frac{t}{2}(0{,}25h^2 - y^2)$$

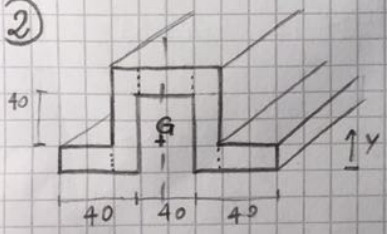
FLUSSO: $q(y) = \frac{V \cdot Q}{I} = \frac{P \cdot t/2 \cdot (0{,}25h^2 - y^2)}{t/12\,(10^{-3} + h^3)}$ → somma tutti i contributi →

$$\rightarrow V_1 = \int_{-h/2}^{+h/2} q(y)\,dy = \frac{6P}{(10^{-3}+h^3)} \int_{-h/2}^{+h/2} (0{,}25h^2 - y^2)\,dy = \frac{6P}{(10^{-3}+h^3)}\left[\frac{1}{4}h^3 - \frac{1}{12}h^3\right] =$$

$$= \frac{Ph^3}{(10^{-3}+h^3)} = 0{,}833 \cdot \cancel{P}$$

$$\rightarrow h^3(1 - 0{,}833) = 0{,}833 \cdot 10^{-3} \rightarrow h = \sqrt[3]{\frac{0{,}833 \cdot 10^{-3}}{1 - 0{,}833}} = 0{,}171\ mm$$

(2)



DATI: $V = 150\,N$ $\quad t = 10\,mm$

INC: max FLUSSO DI TAGLIO

Calcolo G:

$$\bar{y} = \frac{2\left[0{,}005(30)(0{,}01)\right] + 2\left[0{,}03(0{,}01)(0{,}06)\right] + \left[0{,}055(0{,}04)(0{,}01)\right]}{A_{tot}} = 0{,}0277\ m$$

$$I = 2\left[\frac{1}{12}0{,}03 \cdot 0{,}01^3 + (0{,}03 \cdot 0{,}01)(0{,}0277 - 0{,}005)^2\right] + 2\left[\frac{1}{12} \cdot 0{,}01 \cdot 0{,}06^3 + (0{,}06 \cdot 0{,}01)(0{,}03 - 0{,}0277)^2\right] + \left[\frac{1}{12} \cdot 0{,}04 \cdot 0{,}01^3 + (0{,}05 \cdot 0{,}01) \cdot (0{,}055 - 0{,}0277)^2\right]$$

$$\rightarrow I = 0{,}9819 \cdot 10^{-6}\ m^4$$

Il flusso è max dove Q_{max} → sull'ASSE BARICENTRICO:

$$Q_{max} = (0{,}055 - 0{,}0277)(0{,}04 \cdot 0{,}01) + 2(0{,}06 - 0{,}0277)(0{,}01) \cdot \left(\frac{0{,}06 - 0{,}0277}{2}\right)$$

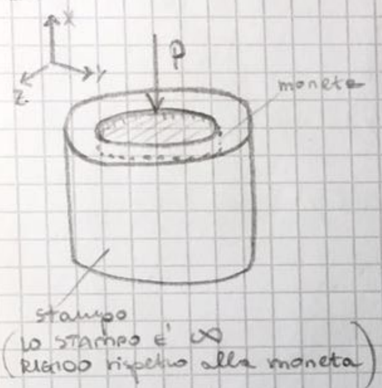
$$= 21{,}3 \cdot 10^{-6}\ m^3$$

$$\rightarrow \Delta V = \frac{\pi}{4}\left((D+\Delta D)^2 - D^2\right)\cdot \Delta L = \frac{\pi}{4}\left(2D\cdot\Delta D + \Delta D^2\right)\Delta L =$$

$$= \frac{\pi}{4}\cdot\left(2\cdot 1000\cdot 3{,}19 + 3{,}19^2\right)(2{,}25) = 1310\ mm^3 = 0{,}0131\ l$$

OSS: La ΔV non è considerevole quanto la Δd!
→ tenerne conto per gli APPOGGI!

5) Ho una MONETA dentro uno STAMPO per realizzare l'EFFIGE



INC: MODULO DI YOUNG APPARENTE: $E' = \frac{\sigma'}{\varepsilon}$

Per via dello stampo → $\varepsilon_{rad} = 0$

$$\begin{cases} \varepsilon_y = 0 \rightarrow & 0 = \sigma_y - \nu(\sigma_x + \sigma_z) \\ \varepsilon_z = 0 \rightarrow & 0 = \sigma_z - \nu(\sigma_x + \sigma_y) \end{cases}$$

$$\rightarrow \begin{cases} \sigma_y = \nu\sigma_x + \nu\sigma_z \\ 0 = \sigma_z - \nu\sigma_x - \nu^2\sigma_x - \nu^2\sigma_z \end{cases}$$

$$\rightarrow \left\{ \sigma_z(1-\nu) = \sigma_x\cdot\nu\cdot(1-\cancel{\nu}) \right.$$

$$\sigma_z = \frac{\nu}{1-\nu}\sigma_x \qquad \sigma_y = \frac{\nu}{1-\nu}\cdot\sigma_x$$

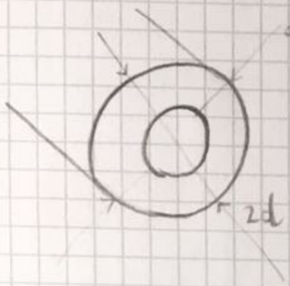
uguali!

$$\rightarrow \varepsilon_x = \frac{1}{E}\left[\sigma_x - 2\nu\sigma_y\right] = \frac{\sigma_x}{E}\left[1 - \frac{2\nu^2}{1-\nu}\right] = \frac{\sigma_x}{E}\left(\frac{1-\nu-2\nu^2}{1-\nu}\right)$$

$$\rightarrow E' = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \simeq 1{,}35$$

→ se costretto, il materiale AUMENTA LA RESIST. A compr. del 35%

5



INC: K ('shape factor')

$$I = \frac{\pi d^4}{64} - \frac{\pi}{4}\left(\frac{d}{2}\right)^4 = \frac{15\pi}{64} d^4$$

$$\sigma_y = \frac{M_y c}{I} \rightarrow M_y = \frac{\sigma_y \cdot I}{\underset{= d}{c}} = \frac{15}{64}\pi d^3 \sigma_y$$

calcolo G $\left(\frac{\text{PIENO}}{\text{VUOTO}}\right)$: $\bar{y} = \frac{\Sigma \bar{y} \cdot A_i}{\Sigma A_i} = \frac{\frac{4}{3}\frac{d}{\pi}\left(\frac{\pi d^2}{2}\right) - \frac{4\, d/2}{3\pi}\left(\frac{\pi}{4}\frac{d^2}{2}\right)}{\frac{\pi d^2}{2} - \frac{\pi}{4}\cdot\frac{d^2}{2}} = \frac{14}{9\pi} d$

della SEMI-CIRCONF.

PLASTICO

$$M_p = \sigma_y \left(\frac{\pi d^2}{2} - \frac{\pi/4\, d^2}{2}\right) \cdot \frac{28}{9\pi} d = \frac{7}{6} d^3 \sigma_y = Z$$

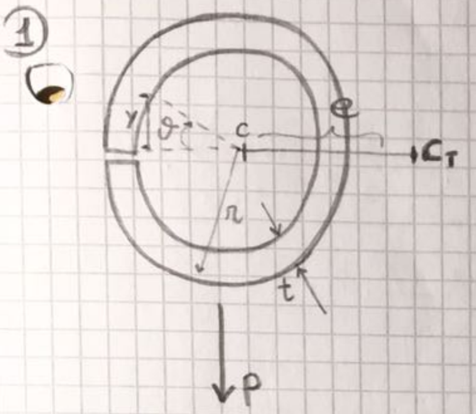
$$\rightarrow K = \frac{M_p}{M_s} = \frac{7}{6} d^3 \cancel{\sigma_y} \frac{64}{15\pi\, d^3 \cancel{\sigma_y}} \simeq 1{,}58$$

8

ESERCITAZIONE 4 (16/01/2017)

1

- DETERMINA C_T -



$$dA = t\,ds = t r\, d\theta \quad , \quad y = r \cdot \sin\theta$$

$$dI = y^2 dA = r^2 \sin^2\theta \cdot t \cdot r\, d\theta$$

$$\rightarrow I = \int_0^{2\pi} dI = r^3 \cdot t \int_0^{2\pi} \sin^2\theta\, d\theta = \text{''} \int_0^{2\pi} \frac{1-\cos(2\theta)}{2}\, d\theta =$$

$$= r^3 t \cdot (\pi)$$

$$dQ = y\,dA = r \cdot \sin\theta \cdot t r\, d\theta$$

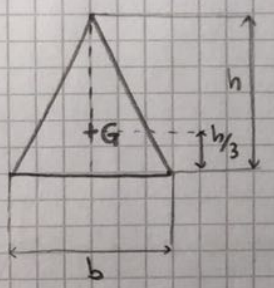
$$\rightarrow Q = r^2 t \int_0^{\theta} \sin\theta\, d\theta = r^2 t\,(1-\cos\theta)$$

$\rightarrow$ flusso: $q = \tau \cdot t = \dfrac{V \cdot Q}{I} = \dfrac{P r^2 t\,(1-\cos\theta)}{r^3 t \cdot \pi} = \dfrac{P}{\pi r}(1-\cos\theta)$ (È IL FLUSSO AD UN CERTO ANGOLO θ)

$$\rightarrow F = \int_0^{2\pi} q \cdot r\, d\theta = \int_0^{2\pi} \frac{P}{\pi \cancel{r}}(1-\cos\theta)\,\cancel{r}\, d\theta = \frac{P}{\pi}\left[2\pi\right] = 2P$$

$\rightarrow$ equil. rotazione: $P \cdot e = F \cdot r \longleftrightarrow \cancel{P} \cdot e = 2\cancel{P} \cdot r \rightarrow \boxed{e = 2r}$

2 - CALCOLA LO 'shape factor' (M_P / M_{sn}):

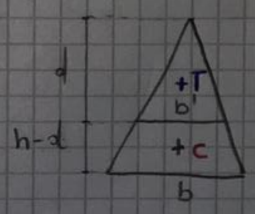


$$\sigma_y = \frac{M_y \cdot c}{I} \rightarrow M_y = \sigma_y \cdot \frac{I}{c}$$

$\boxed{I = \dfrac{1}{36} b h^3}$ (RICORDA!)

$$\rightarrow M_y = \frac{\sigma_y}{36} \cdot b h^3 \cdot \frac{3}{2h} = \frac{1}{24} b h^2 \sigma_y$$

RICORDA: in PLASTICIZZAZIONE su tutta la sezione la $\sigma = \sigma_y$, quindi



$\rightarrow T = C$ (Equilibrio ASSIALE)

$$\rightarrow \cancel{\sigma_y} \cdot A_T = \cancel{\sigma_y} \cdot A_C \rightarrow$$

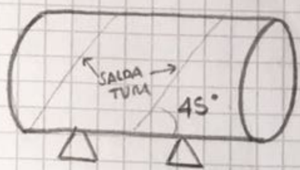
$$\rightarrow \frac{b'd}{2} = \frac{b+b'}{2} \cdot (h-d)$$

sostit.

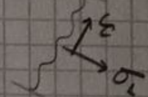
similit. TRIANGOLI: $\dfrac{b'}{d} = \dfrac{b}{h} \rightarrow b' = \dfrac{b \cdot d}{h}$

$$\rightarrow \frac{bd^2}{2h} = \frac{b + \frac{b \cdot d}{h}}{2} \cdot (h-d) = \frac{bh}{2} - \cancel{\frac{bd}{2}} + \cancel{\frac{bd}{2}} - \frac{bd^2}{2h} \longleftrightarrow \frac{\cancel{b}d^2}{h} = \frac{\cancel{b}h}{2} \longrightarrow$$

③ - SERBATOIO CILINDRICO - è una PIASTRA AVVOLTA A SPIRALE



INC: stato di tensione sul CORDONE DI SALDATURA



hp: $\frac{r}{t} = \frac{1250}{15} \simeq 83$ → ST. TENS. PIANO

DATI: $r = 1{,}25$ m
$t = 15$ mm
$p_i = 30$ atm
$\simeq 0{,}1 \cdot 30$ MPa
$= 3$ MPa

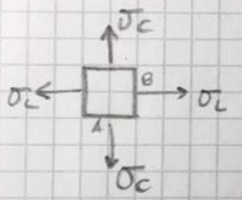
→ $\sigma_L = \frac{pr}{2t}$, $\sigma_c = \frac{pr}{t}$
(LONGITUDINALE) (CIRCONFERENZIALE)

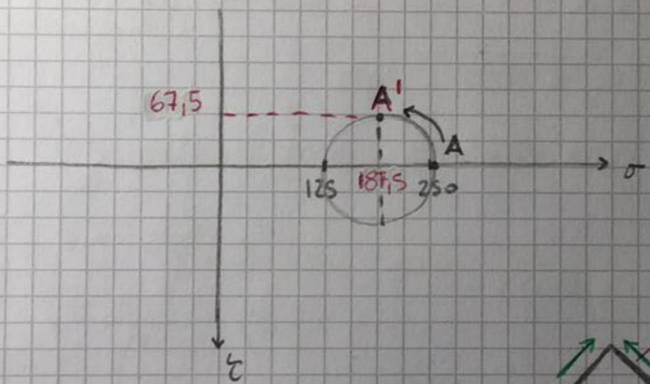
$\sigma_L = \frac{3 \cdot 1250}{2 \cdot 15} = 125$ MPa , $\sigma_c = 250$ MPa

↦ $\sigma(\alpha) = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\alpha + \tau_{xy} \sin 2\alpha$

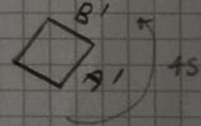
$\tau(\alpha) = \tau_{xy} \cos(2\alpha) - \frac{1}{2}(\sigma_x - \sigma_y)\sin(2\alpha)$

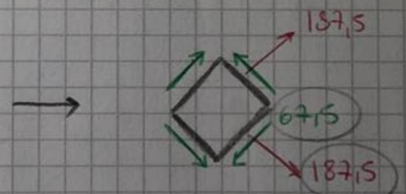
Circ. MOHR





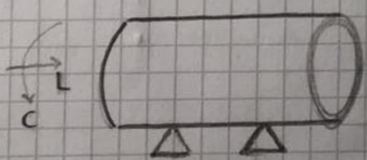
→ ruoto A di 45°
→ A'





È VUOTO → LO RIEMPIO CON UN GAS

④ - SERBATOIO CILINDRICO - Da scarico a carico quanto AUMENTA V?



$\sigma_c = \frac{pR}{t}$ $\sigma_L = \frac{pR}{2t}$

$\varepsilon_c = \frac{1}{E}\left[\sigma_c - \nu\sigma_L\right] = \frac{pR}{2Et}(2-\nu)$

$\varepsilon_L = \frac{1}{E}\left[\sigma_L - \nu\sigma_c\right] = \frac{pR}{2Et}(1-2\nu)$

DATI: $R = 0{,}5$ m
$L = 3$ m
$t = 10$ mm
$p = 15$ MPa
$E = 200$ GPa
$\nu = 0{,}3$

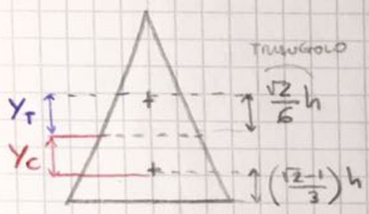
→ LA VARIAZIONE DEL DIAMETRO : $\Delta d = \varepsilon_c \cdot d = \frac{15 \cdot 10^6 \cdot 0{,}5}{2 \cdot 200 \cdot 10^9 \cdot 0{,}001} \cdot (2 - 0{,}3) \cdot 1000 =$

$= \underline{3{,}19 \text{ mm}}$

$\Delta V = ?$ → LA VARIAZIONE DI L : $\Delta L = \varepsilon_L \cdot L = \frac{15 \cdot 10^6 \cdot 0{,}5}{2 \cdot 200 \cdot 10^9 \cdot 0{,}01} \cdot (1 - 0{,}6) \cdot 3000 = \underline{2{,}25 \text{ mm}}$

$\rightarrow \underline{d} = \frac{\sqrt{2}}{2} h \quad , \quad \underline{h-d} = h - \frac{\sqrt{2}}{2} h = \frac{2-\sqrt{2}}{2} h$ (sono le 2 ALTEZZE cercate!)

Calcolo i due baricentri:



Per il TRAPEZIO la formula è: $\boxed{Y_G^{TR} = \frac{B+2b}{B+b} \cdot \frac{H}{3}}$

$$\rightarrow Y_{GT} = \frac{2-\sqrt{2}}{6} h \cdot \frac{\cancel{b}+\sqrt{2}\cancel{b}}{\cancel{b}+\frac{\sqrt{2}}{2}\cancel{b}} = \frac{2-\sqrt{2}}{\cancel{6}\,3} \cdot h \cdot \cancel{2} \cdot \frac{1+\sqrt{2}}{2+\sqrt{2}} =$$

RAZIONALIZZO

$$= \frac{2-\sqrt{2}}{3} \cdot h \cdot \frac{1+\sqrt{2}}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}} =$$

$$= \frac{(4+2-4\sqrt{2})(1+\sqrt{2})}{3 \cdot 2} h = \frac{(6-4\sqrt{2})(1+\sqrt{2})}{6} =$$

$$= \frac{6-4\sqrt{2}+6\sqrt{2}-8}{6} h = \frac{2\sqrt{2}-2}{6} h = \left(\frac{\sqrt{2}-1}{3}\right) h$$

$$\rightarrow Y_C = \frac{2-\sqrt{2}}{2} h - \frac{\sqrt{2}-1}{3} h = \frac{6-3\sqrt{2}-2\sqrt{2}+2}{6} h =$$

$$= \frac{8-5\sqrt{2}}{6} h$$

$$Y_T = \frac{\sqrt{2}}{6} h$$

$$\rightarrow M_p = \sigma_y \left[\frac{1}{2}\left(\frac{\sqrt{2}}{2} b\right)\left(\frac{\sqrt{2}}{2} h\right)\right] \cdot \left[\frac{\sqrt{2}}{6} h + \frac{8-5\sqrt{2}}{6} h\right] =$$

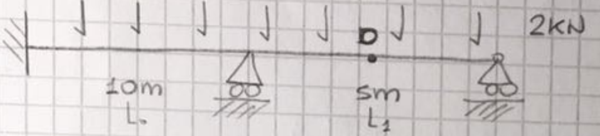
$$= \sigma_y \left(\frac{bh}{4}\right)\left(\frac{4-2\sqrt{2}}{3} h\right) = \sigma_y \frac{4-2\sqrt{2}}{12} bh^2$$

$\rightarrow$ 'shape factor': $K = \frac{M_p}{M_y} = \frac{2-\sqrt{2}}{6} \cdot 24 = 8-4\sqrt{2} = 2{,}34$

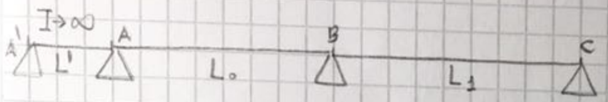
$\rightarrow$ posso sovraccaricare del 234% prima del collasso!

$\rightarrow$ la punta plasticizza subito, converrebbe troncarla

3) Trave su multipli appoggi



$\delta_D = ?$



Equazione dei 3 momenti : $M_0 \frac{L_0}{I_0} + 2M_1\left(\frac{L_0}{I_0} + \frac{L_1}{I_1}\right) + \frac{M_1 L_1}{I_1} = -6E\left(\vartheta_0^{(1)} + \vartheta_1^{(1)}\right)$

$[A'B]$: $0 \cdot \frac{L'}{\infty} + 2M_A\left(\frac{L'}{\infty} + \frac{10}{I}\right) + M_B \cdot \frac{10}{I} = -6E\left(\cancel{\vartheta_{A'A}} + \vartheta_{AB}\right)$

So che $\vartheta_{AB} = \frac{qL^3}{24EJ}$

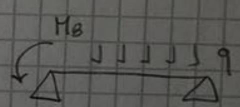
$\rightarrow \underbrace{20 M_A + 10 M_B = -6 \cdot 8{,}33 \cdot 10^4 = -5 \cdot 10^5}$ (I)

$M_A\left(\frac{10}{I}\right) + 2M_B\left(\frac{10}{I} + \frac{5}{I}\right) = -6E\left(\frac{8{,}33 \cdot 10^4}{EJ} + \frac{1{,}042 \cdot 10^4}{EJ}\right)$

$\rightarrow \underbrace{10 M_A + 30 M_B = -5{,}625 \cdot 10^5}$ (II)

$\rightarrow$ sistema (I) + (II) : $\begin{cases} M_A = -18{,}75 \text{ kN}\cdot\text{m} \\ M_B = -12{,}5 \text{ kN}\cdot\text{m} \end{cases}$

$\rightarrow$ La soluz. è data da sovrapposizione di $q + M_B$:

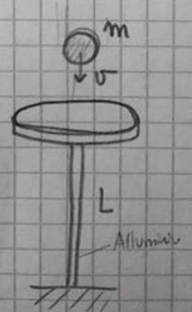


$v\left(\frac{BC}{2}\right) = -\frac{5}{384} \cdot \frac{qL^4}{EJ} - \frac{M_B \cdot L^2}{16EJ} = \frac{35807}{EJ}$

4) Instabilita'

DATI : $m = 20$ kg $\quad L = 1$m $\quad d_e = 100$ mm $\quad s = 6$ mm

$E_{All} = 72000$ MPa $\quad \nu = 0{,}29 \quad \sigma_y = 210$ MPa (di flusso plastico)



INC : 1) - v che manda in INSTABILITA' (URTO CENTRATO)

2) - v (URTO DECENTRATO) $\delta = 120$ mm

3) - $\delta_{LATERALE}$ nel caso 2

$K = \frac{EA}{L} = \frac{72000 \cdot 10^6 \cdot \pi\,(0{,}100^2 - 0{,}088^2)}{1 \cdot 4} = 1{,}27 \cdot 10^8$ N/m

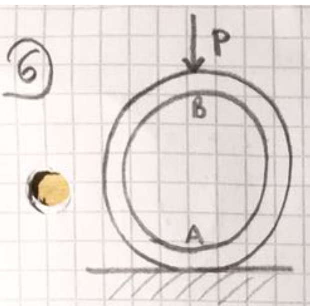
$\rightarrow$ conserv. ENERGIA : $\frac{1}{2}mv^2 = \frac{1}{2}K\Delta^2 \rightarrow \Delta = \sqrt{\frac{mv^2}{K}} = 3{,}97 \cdot 10^{-4}\, v$ [m]

$\rightarrow F = K \cdot \Delta = 1{,}27 \cdot 10^8 \cdot 3{,}97 \cdot 10^{-4}\, v = 50'419\, v$ [N]

1) $P_{cr} = \frac{\pi^2 EJ}{L_{eq}^2} = \frac{\pi^2 EJ}{4L^2} = \frac{\pi^2 \cdot 72000 \cdot 10^6 \cdot \pi\,(0{,}100^4 - 0{,}088^4)}{64 \cdot 4 \cdot 1^2} = 349'000$ [N]

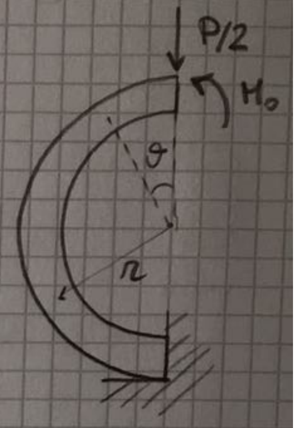
$\rightarrow v = P_{cr} / F = 6{,}92$ m/s $\rightarrow$ VERIFICHE :

6



- CALCOLA δ_B (STRUTTURA IPERSTATICA) -

considero un '<u>semi-anello</u>': e metto un M_o, il momento che si scambiano i 2 semi-anelli =



$$M(\theta) = \frac{P}{2} \cdot r \sin\theta - M_o$$

→ APPLICO CASTIGLIANO alla condiz. vincolare : $\left.\dfrac{\partial M}{\partial M_o}\right|_{\theta=0} = 0$

↳ $\dfrac{1}{EI}\displaystyle\int M \left(\frac{\partial M}{\partial M_o}\right) ds \overset{\downarrow}{=} \phi$ (LA ROTAZ. E' NULLA)

→ $\displaystyle\int_0^\pi \left(\frac{Pr}{2}\sin\theta - M_o\right)(-1)\, r\, d\theta \overset{\downarrow}{=} 0$

→ $\dfrac{Pr^{\cancel{2}}}{2}(2) - M_o \cdot \cancel{r}\pi = 0$ → $M_o = \dfrac{Pr}{\pi}$ → noto M_o il problema è ISOSTATICO

→ <u>Riapplico</u> CASTIGLIANO: $\Delta = \displaystyle\int_0^L M \frac{\partial M}{\partial P} \frac{ds}{EI}$

→ $\Delta = \dfrac{1}{EI}\displaystyle\int_0^\pi \left(\frac{Pr}{2}\sin\theta - \frac{Pr}{\pi}\right) r\sin\theta \cdot r\, d\theta =$

$= \dfrac{1}{EI} \cdot \left[\dfrac{Pr^3}{2\pi}\right]\left(\displaystyle\int_0^\pi (\pi\sin^2\theta - 2\sin\theta)\, d\theta\right) =$

$= \dfrac{1}{EI} \cdot \dfrac{Pr^3}{4\pi} \displaystyle\int_0^\pi \pi\,(1-\cos(2\theta) - 4\sin\theta)\, d\theta =$

$= \dfrac{Pr^3}{4\pi EI}\left(\pi^2 - 8\right)$

→ <u>OSS</u>: - La prima volta APPLICO per risolvere la condizione vincolare
- La seconda volta " " lo SPOSTAMENTO

• Devo però verificare di non uscire dal LIMITE DI ELASTICITA' (l'ho ipotizzata ovunque)

$$\rightarrow \quad \sigma = \frac{P_{cr}}{A} = \frac{349\,000}{\pi(0{,}100^2 - 0{,}088^2)} \cdot 4 = 197 \text{ MPa} < 210 \text{ MPa} \quad \checkmark$$

• Devo verificare di NON avere INSTABILITA' LOCALE:

$$\Delta = \frac{2\pi}{\sqrt[4]{12(1-\nu^2)}}\sqrt{RS} = 0{,}013 \text{ m} \quad \checkmark$$

• $$\sigma_{CR} = \frac{E}{\sqrt{3(1-\nu^2)}}\left(\frac{S}{R}\right) = 5923 \text{ MPa} \quad \checkmark$$

E' ALTA, QUINDI DI SICURO AVVERRA' PRIMA LO SNERVAMENTO, CHE CMQ HO SCONGIURATO

2) - (σ_Y)

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{e \cdot c}{\rho^2} \cdot \sec\left(\frac{L}{2 \cdot \rho}\sqrt{\frac{P}{EA}}\right)\right] \qquad \text{con} = \quad P = F \cdot v$$

→ L'INCOGNITA E' v, ma l'EQ. E' NON LINEARE → risolvo x ITERAZIONE

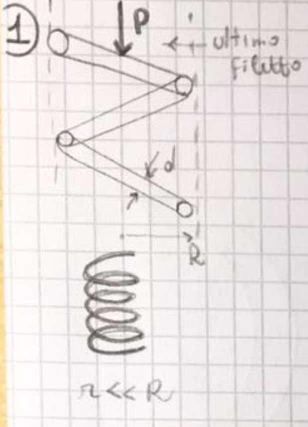
v	σ_{max}
1	45,6
5	228,4
4	182,7
→ 4,6	210

→ $v = 4{,}6$ m/s

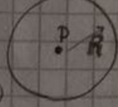
3) - $$\delta = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} - 1\right)\right] =$$

$$= 0{,}120 \cdot \left[\sec\left(\frac{\pi}{2}\sqrt{\frac{50419 \cdot 4{,}6}{349\,000}} - 1\right)\right] = 4{,}88 \text{ mm}$$

27-01-2016 - ESERCITAZIONE 5

1) 

- Valuta la RIGIDEZZA della molla - $k = \frac{P}{\Delta}$ - dall'alto -



La spirale e' soggetta a TORSIONE COSTANTE
(trascuro le altre sollecitaz.)
FLESS. e TAGLIO

$$M(\vartheta) = P \cdot R = \text{cost.}$$

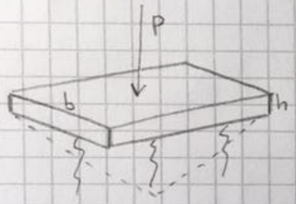
$$U = \underset{n\,spire}{n} \cdot \int_0^{2\pi} \frac{1}{2} \frac{M^2}{GJ_p} R\,d\vartheta = n \int_0^{2\pi} \frac{P^2R^3}{2GJ_p} d\vartheta = \frac{nP^2R^3\,2\pi}{2GJ_p} =$$

$$= \frac{32\,P^2R^3n}{Gd^4}$$

→ applico Castigliano per trovare lo spostamento:

$$\Delta = \frac{\partial U}{\partial P} = \frac{64 \cdot PR^3n}{Gd^4} \rightarrow k = \frac{P}{\Delta} = \frac{G \cdot d^4}{64R^3n}$$

2) - Calcola K della fondazione tale da avere def. PLASTICA nella billetta



Dati:
$P = 1500\,N$
$\sigma_y = 250\,MPa$
$L = 300\,mm$
$h = 5\,mm$
$b = 10\,mm$

Travi su fondaz. elastica

$$I = \frac{1}{12}bh^3 = 104{,}17\ mm^4$$

$$M_P = M_S \cdot \frac{3}{2} = \frac{bh^2}{6} \cdot \frac{3}{2} = 15625\ N \cdot m$$

(mi serve PLASTICIZZ. TOTALE!)

costante

$$\eta(x) = F\,e^{-\beta x}\left(\cos(\beta x) + \sin(\beta x)\right)$$

$$\frac{d\eta}{dx} = -2F\,e^{-\beta x}\sin(\beta x)$$

$$\frac{d^2\eta}{dx^2} = 2\beta^2F\,e^{-\beta x}\left(\sin(\beta x) - \cos(\beta x)\right)$$

$$\frac{d^3\eta}{dx^3} = 4\beta^3F\,e^{-\beta x}\cos(\beta x)$$

(mi serve derivare fino a qua per avere B.C. con il TAGLIO)

Applico le B.C.: $-\frac{P}{2} = -EI \left.\frac{d^3\eta}{dx^3}\right|_{x=0} = -4\beta^3F \cdot EI \rightarrow F = \frac{P}{8EI\beta^3}$

il Mmax e' al centro

$$\rightarrow M_{max}\Big|_{x=0} = -EI \left.\frac{d^2\eta}{dx^2}\right|_{x=0} = -\frac{P}{4\beta} e^{-\beta x}\left(\cos(\beta x) + \sin(\beta x)\right)\Big|_{x=0} = -\frac{P}{4\beta} = M_p$$

uso del diagramma del mom.

$$\beta = \sqrt[4]{\frac{K}{4EJ}} \rightarrow K = 4EJ\left(-\frac{P}{4M_p}\right)^4 =$$

$$= 4 \cdot 2{,}06 \cdot 10^{11} \cdot 104{,}17 \cdot 10^{-12} \cdot \left(-\frac{150}{4 \cdot 15625}\right)^4 = 2847\ \frac{N}{m^2}$$