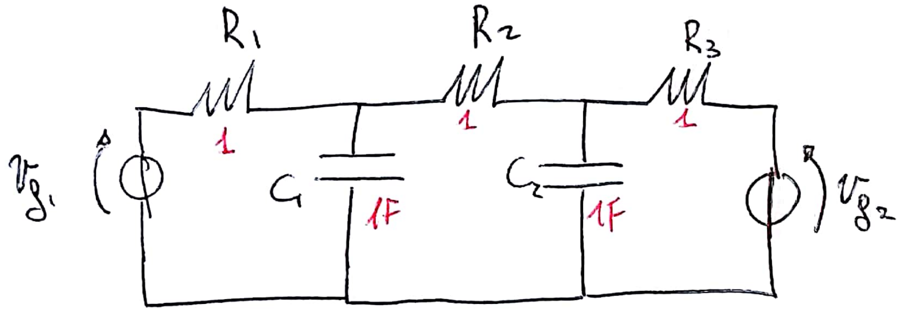
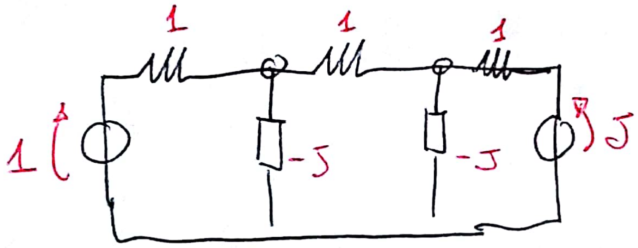
Ex 4



$v_{g_1}(t) = \sin(\omega t)$ $\qquad \overline{V_{g_1}} = 1$

$v_{g_2}(t) = \cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$ $\qquad \overline{V_{g_2}} = 1\left(\cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)\right) = j$

$Z_{C_1} = -j \qquad Z_{C_2} = -j \qquad Y_{C_1} = j \qquad Y_{C_2} = j$



$$\begin{pmatrix} 2+j & -1 \\ -1 & 2+j \end{pmatrix} \begin{pmatrix} \overline{V_1} \\ \overline{V_2} \end{pmatrix} = \begin{pmatrix} 1 \\ j \end{pmatrix}$$

$\det : (2+j)(2+j) - 1$

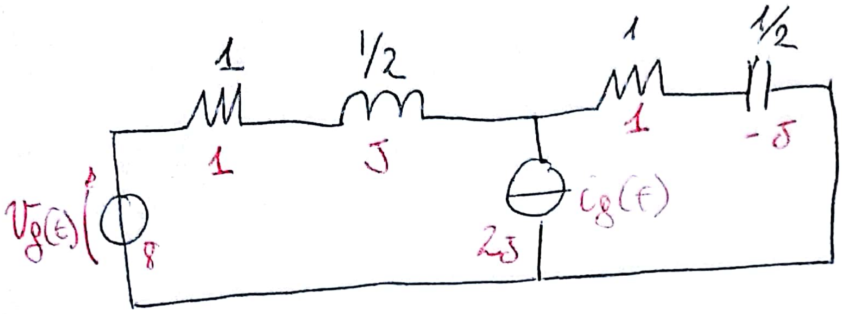
$= 4 + 2j + 2j - 1 - 1 = 2 + 4j$

$$\frac{1}{2+4j} \begin{pmatrix} 2+j & 1 \\ 1 & 2+j \end{pmatrix} \begin{pmatrix} 1 \\ j \end{pmatrix} = \begin{pmatrix} \overline{V_1} \\ \overline{V_2} \end{pmatrix}$$

$\overline{V_1} = \frac{3}{5} - \frac{1}{5}j$

$\overline{V_2} = \frac{2}{5} + \frac{1}{5}j$

Ex Nuovo 2



$i_g(t) = 2\cos(2t)$

$V_g(t) = 8\sin(2t)$

$i_g(t) = 2\sin(2t + \frac{\pi}{2})$

$\bar{V}_g = 8 \quad \bar{I}_g = 2J$

Nodi

$$\left(\frac{1}{1+J} + \frac{1}{1-J}\right)\bar{V}_x = \frac{8}{1+J} + 2J$$

$$\left(\frac{1-J+1+J}{\cancel{2}\ 1}\right)\bar{V}_x = \frac{8+2J(1+J)}{1+J} = \frac{8+2J-2}{1+J} = \frac{2J+6}{1+J}$$

$$\bar{V}_x = \frac{2J+6}{1+J} = \frac{(2J+6)(1-J)}{2} = \frac{2J+2+6-6J}{2} = 4-2J$$

$$S_{I_g} = \frac{1}{2}\bar{V}\bar{I}^* = \frac{1}{2}(\bar{V}_x)(\bar{I}_g)^* = \frac{1}{2}(4-2J)(-2J) =$$

$$= -4J-2 \begin{cases} \to P = -2W \\ Q = -4\,VAR \end{cases}$$

Ex 1

$v_{g_1} = \cos(t) \to \omega = 1 \text{ rad/s}$

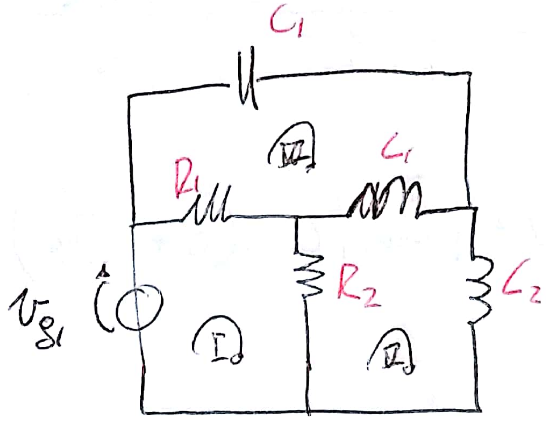
$\to \bar{V}_1 = 1$

$C_1 = 1F \to Z_{C_1} = -j$

$L_1 = 1H \to Z_{L_1} = j$

$L_2 = 2H \to Z_{L_2} = 2j$

$R_1 = R_2 = 2\Omega \qquad Z_R = 2$



$$\begin{bmatrix} Z_{R_1}+Z_{R_2} & -Z_{R_2} & -Z_{R_1} \\ -Z_{R_2} & Z_{R_2}+Z_{L_1}+Z_{L_2} & -Z_{L_1} \\ -Z_{R_1} & -Z_{L_1} & Z_{C_1}+Z_{L_1}+Z_{R_1} \end{bmatrix} \begin{bmatrix} \bar{I}_{m_1} \\ \bar{I}_{m_2} \\ \bar{I}_{m_3} \end{bmatrix} = \begin{bmatrix} \bar{V}_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{pmatrix} 4 & -2 & -2 \\ -2 & (2+3j) & -j \\ -2 & -j & 2 \end{pmatrix} \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

det: $(16+24j-4j-4j)-(8+12j-4+8) = 4+4j$

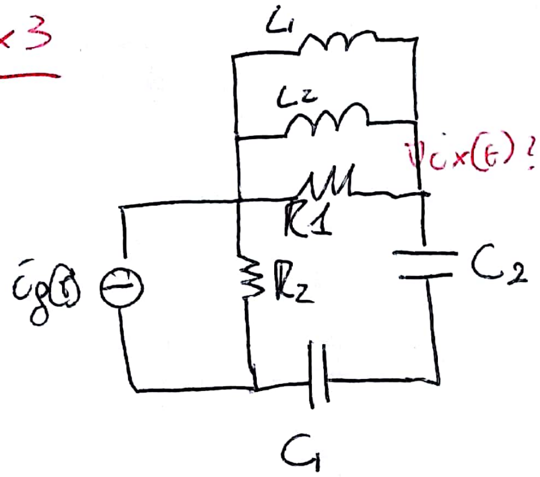
$c_1)$ $\begin{vmatrix} 1 & -2 & -2 \\ 0 & (2+3j) & -j \\ 0 & -j & 2 \end{vmatrix} = (4+6j)-(-1) = 5+6j$

$$\bar{I}_{m_1} = \frac{5+6j}{4+4j} = \frac{(5+6j)(4-4j)}{32} = \frac{44}{32} + \frac{4}{32}j = \frac{11}{8} + j\frac{1}{8}$$

$$i_s(t) = |\bar{I}_{m_1}| \cos(t + \angle \bar{I}_{m_1}) \qquad |\bar{I}_{m_1}| = \sqrt{\left(\frac{11}{8}\right)^2 + \left(\frac{1}{8}\right)^2}$$

$$\angle \bar{I}_{m_1} = tg^{-1}\left(\frac{1}{11}\right)$$

Ex3



$i_g(t) = 2\cos(t)$ $\omega = 1$

$\bar{I}_g = 2$

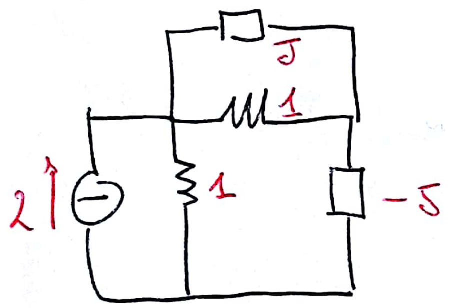
$L_2 = L_1 = 2H$ $Z_{L_1} = Z_{L_2} = 2j$

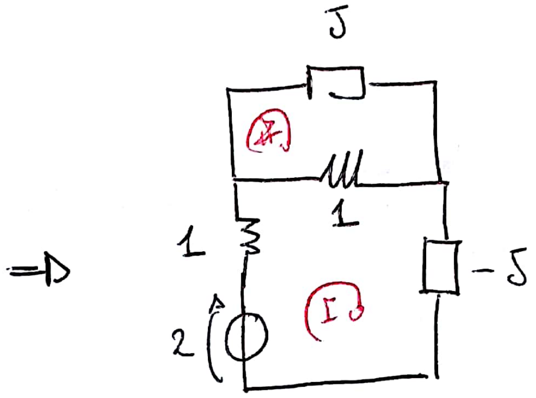
$C_1 = C_2 = 2F$ $Z_{C_1} = Z_{C_2} = -j/2$

$R_1 = R_2 = 1\Omega$

$$Z_{L_{TOT}} = Z_{L_1} \parallel Z_{L_2} = \frac{2j \cdot 2j}{2j + 2j} = \frac{-4}{4j} = -\frac{1}{j} = \boxed{j}$$

$$Z_{C_{TOT}} = Z_{C_1} + Z_{C_2} = -j$$





Maglie

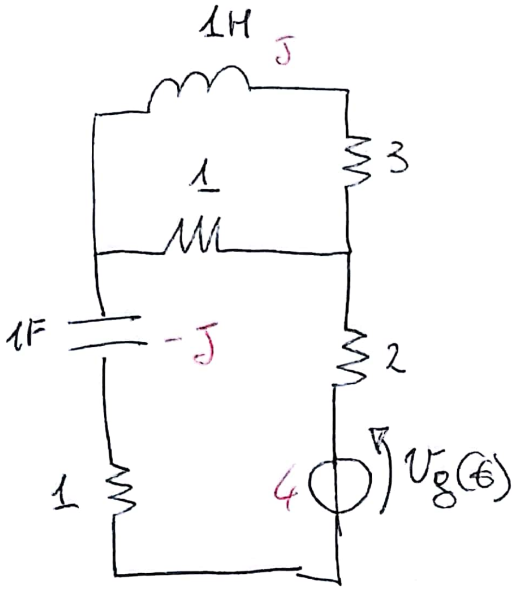
$$\begin{bmatrix} (2-j) & -1 \\ -1 & 1+j \end{bmatrix} \begin{bmatrix} \bar{I}_{m_1} \\ \bar{I}_{m_2} \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\det: \left[(2-j)(1+j)\right] - 1 =$$
$$= 2 + 2j - j + 1 - 1 = 2 + 2j - j$$
$$= 2 + j$$

$\bar{I}_{m_2}$ $$\begin{vmatrix} 2-j & 2 \\ -1 & 0 \end{vmatrix} = 2$$

$$\bar{I}_{m_2} = \frac{2}{2+j} = \frac{4}{5} - \frac{2}{5}j$$

Ex Nuovo



$$V_g(t) = 4\cos(t)$$

Maglie

$$\begin{bmatrix} 4-j & -1 \\ -1 & 4+j \end{bmatrix} \begin{bmatrix} \bar{I}_{m_1} \\ \bar{I}_{m_2} \end{bmatrix} = \begin{bmatrix} -4 \\ 0 \end{bmatrix}$$

det: 16

$$I_{m_1} = -1 - \frac{1}{4}j$$

$$I_{m_2} = -\frac{1}{4}$$

Generatore

$$S = \frac{1}{2}\bar{V}\bar{I}^* = \frac{1}{2}(4)(-I_{m_1})^* = 2\left(1 - \frac{1}{4}j\right) = 2 - \frac{1}{2}j \quad \begin{cases} \to P = 2W & \text{EROG.} \\ Q = -\frac{1}{2}\,\text{VAR} & \text{ASS.} \end{cases}$$

Capacità

$$Q = \frac{1}{2}\,\text{Im}\{Z_C\}|\bar{I}|^2 = \frac{1}{2}\,\text{Im}\Big\{\overbrace{-\frac{j}{\omega C}}^{-1}\Big\}|\bar{I}|^2 = \qquad |\bar{I}|^2 = \frac{17}{16}$$

$$= \frac{1}{2}(-1)\frac{17}{16} = -\frac{17}{32}\,\text{VAR} \quad \text{EROG.}$$

Induttanza

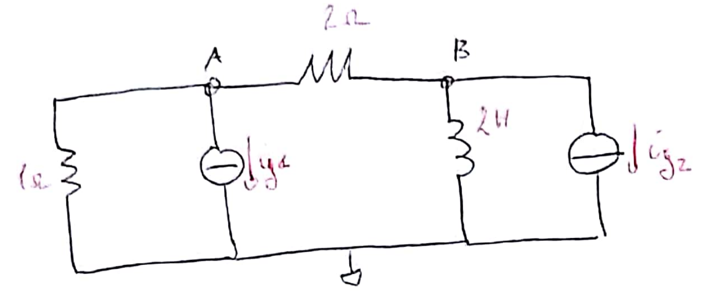
$$Q = \frac{1}{2}\,\text{Im}\{Z_L\}|\bar{I}|^2 = \frac{1}{2}(1)\cdot\frac{1}{16} = \frac{1}{32}\,\text{VAR} \quad \text{ASS.}$$

$$-\frac{17}{32} + \frac{1}{32} = -\frac{16}{32} = -\frac{1}{2}$$

Pot. Reattiva ASSORBITA DA V_g è uguale a Pot. Reattiva erogata

$\boxed{\text{Es } 1}$

$i_{g_1} = i_{g_2} = 2\cos(2t + \frac{\pi}{4})$



P, Q su i_{g_1} ?

$Y_L = \frac{1}{j\omega L} = -\frac{j}{4}$

$\bar{I}_{g_1} = \bar{I}_{g_2} = 2$

Nodi

$$\begin{pmatrix} 3/2 & -1/2 \\ -1/2 & \frac{1}{2} - \frac{j}{4} \end{pmatrix} \begin{pmatrix} \bar{V}_A \\ \bar{V}_B \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

$$\text{det}: \frac{3}{4} - \frac{3}{8}j - \frac{1}{4} = \frac{6 - 3j - 2}{8} = \frac{2}{4} - \frac{3}{8}j = \frac{4 - 3j}{8}$$

$$\begin{vmatrix} -2 & -1/2 \\ -2 & \frac{1}{2} - \frac{j}{4} \end{vmatrix} = -1 + \frac{j}{2} - 1 = \frac{j}{2} - 2 = \frac{j - 4}{2}$$

$$\bar{V}_A = \frac{(j-4)\cdot 8}{(4-3j)\cdot 2} = \frac{4j - 16}{4 - 3j} = \frac{(4j-16)(4+3j)}{25} = \frac{16j - 64 - 12 - 48j}{25}$$

$$= -\frac{32}{25}j - \frac{76}{25}$$

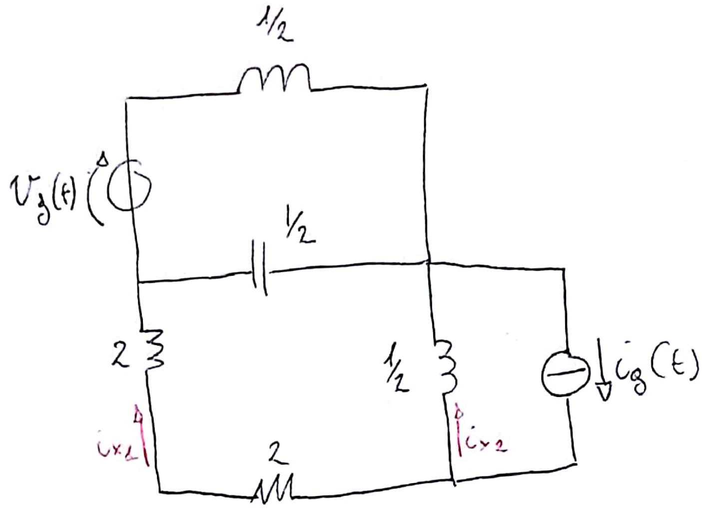
$$P = \frac{1}{2}\mathrm{Re}\{\bar{V}\bar{I}^*\} = \frac{1}{2}\mathrm{Re}\left\{\left[-\frac{32}{25}j - \frac{76}{25}\right][2]\right\} = \boxed{-\frac{76}{25}\,\text{W}}$$

$$Q = \frac{1}{2}\mathrm{Im}\{\bar{V}\bar{I}^*\} = \frac{1}{2}\mathrm{Im}\left\{\left[-\frac{32}{25}j - \frac{76}{25}\right][2]\right\} = \boxed{-\frac{32}{25}\,\text{VAR}}$$

Ex 2

$v_g(t) = \cos(2t)$

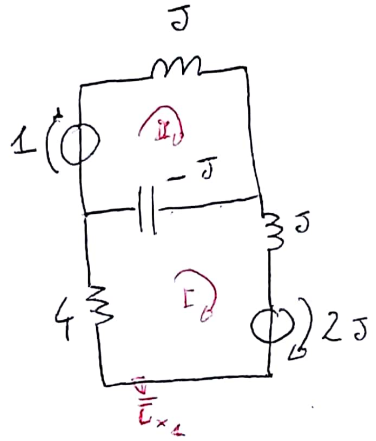
$i_g(t) = 2\cos(2t)$



$\bar{V}_g = 1 \quad \bar{I}_g = 2$

$Z_L = j \quad Z_C = -j$

Fasori + Nort. → Thev.



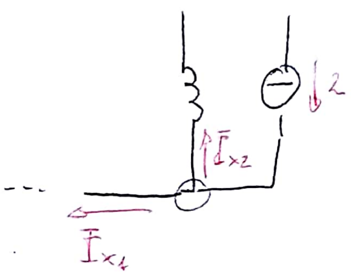
$$\begin{pmatrix} 4 & j \\ j & 0 \end{pmatrix} \begin{pmatrix} \bar{I}_{m1} \\ \bar{I}_{m2} \end{pmatrix} = \begin{pmatrix} 2j \\ 1 \end{pmatrix}$$

det = 1

$$\begin{vmatrix} 2j & j \\ 1 & 0 \end{vmatrix} = -j$$

$\boxed{\bar{I}_{x1} = -j}$

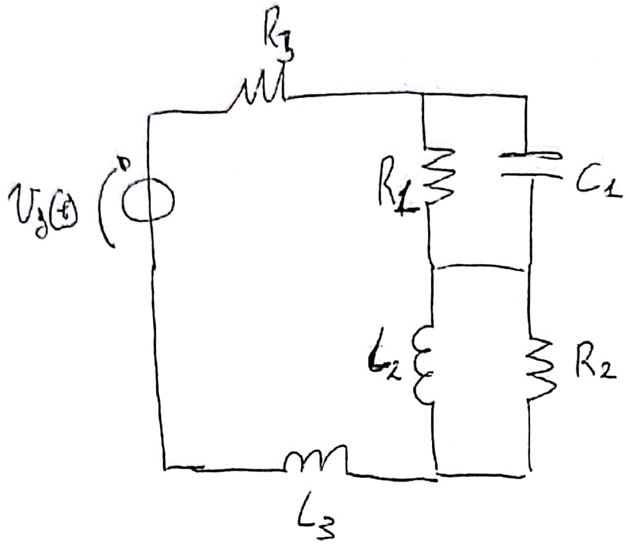
Per I_{x2} ho



$$2 - \bar{I}_{x2} - \bar{I}_{x1} = 0 \Rightarrow \bar{I}_{x2} = 2 - \bar{I}_{x1} = 2 + j$$

$$Q_L = \frac{1}{2} \mathrm{Im}\{Z_L\} |\bar{I}|^2 = \frac{1}{2} \cdot (1) \cdot 5 = \frac{5}{2} \text{ VAR}$$

Ex 3



$V_g(t) = 5\sin(4t)$

$R_1 = 2 \to Z_{R_1} = 2$

$R_2 = 2 \to Z_{R_2} = 2$

$R_3 = 1 \to Z_{R_3} = 1$

$C_1 = 1/4 \to Z_{C_1} = -j$

$L_1 = 1H \to Z_{L_1} = 4j$

$L_2 = 1H \to Z_{L_2} = 4j$

$$Z_{TOT} = Z_{R_3} + \left(Z_{R_1} /\!/ Z_{C_1}\right) + \left(Z_{L_2} /\!/ Z_{R_2}\right) + Z_{L_3}$$

$$Z_{R_1} /\!/ Z_{C_1} = \frac{2\cdot(-j)}{2-j} = \frac{-2j(2+j)}{5} = -\frac{4}{5}j + \frac{2}{5}$$

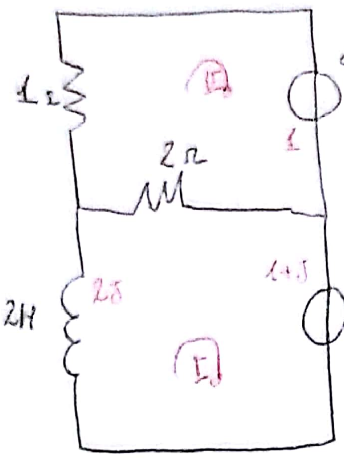
$$Z_{L_2} /\!/ Z_{R_2} = \frac{2\cdot(4j)}{2+4j} = \frac{\overset{2}{\cancel{8}}j(2-4j)}{\cancel{20}\,5} = \frac{\cancel{4}}{5}j + \frac{8}{5}$$

$$Z_{TOT} = 1 + \frac{4}{5}j + \frac{8}{5} - \frac{4}{5}j + \frac{2}{5} + 4j = 3 + 4j$$

$$Y_{TOT} = \frac{1}{3+4j} = \frac{3-4j}{25} = \frac{3}{25} - \frac{4}{25}j$$

$$S_{V_g} = \frac{1}{2}\bar{V}\bar{I}^* = \frac{1}{2}|\bar{V}|^2 Y_{TOT}^* = \frac{1}{2}\cdot(25)\cdot\left(\frac{3}{25} + \frac{4}{25}j\right) = \underbrace{\frac{3}{2}}_{P} + \underbrace{2j}_{Q}$$

Ex 4



$v_{g1}(t) = \cos(t) \Rightarrow \bar{V}_{g1} = 1$

$v_{g2}(t) = \sqrt{2}\cos(t + \pi/4) \Rightarrow \bar{V}_{g2} = \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = 1 + j$

Maglie

$$\begin{pmatrix} 2j+2 & -2 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} \bar{I}_{m_1} \\ \bar{I}_{m_2} \end{pmatrix} = \begin{pmatrix} -1-j \\ -1 \end{pmatrix}$$

$\det: 6j + 6 - 4 = 6j + 2$

$$\begin{vmatrix} -1-j & -2 \\ -1 & 3 \end{vmatrix} = -3 - 3j - 2 = -5 - 3j \Rightarrow \bar{I}_{m_1} = \frac{-5-3j}{6j+2} = \frac{-5-3j(2-6j)}{40} =$$

$$= \frac{-10 + 30j - 6j - 18}{40} = -\frac{28}{40} + \frac{24}{40}j =$$

$$= -\frac{7}{10} + \frac{6}{10}j$$

$$\begin{vmatrix} 2j+2 & -1-j \\ -2 & -1 \end{vmatrix} = -2j - 2 - 2 - 2j = -4j - 4$$

$$\Rightarrow \bar{I}_{m_2} = \frac{-4j-4}{6j+2} = \frac{(-4j-4)(2-6j)}{40} =$$

$$= \frac{-8j - 24 - 8 + 24j}{40} = -\frac{32}{40} + \frac{16}{40}j$$

$$= -\frac{4}{5} + \frac{2}{5}j$$

$\cancel{S_{v_{g1}} = \frac{1}{2}(1)\left(-\frac{7}{10} - \frac{6}{10}j\right) = \boxed{-\frac{7}{20}}\ \boxed{-\frac{6}{20}}j}$

P Q

$\cancel{S_{v_{g2}} = \frac{1}{2}(1+j)\left(-\frac{4}{5} - \frac{2}{5}j\right) = \left(-\frac{4}{5} - \frac{2}{5}j - \frac{4}{5}j + \frac{2}{5}\right)/2 =}$

$\cancel{= \left(-\frac{2}{5} - \frac{6}{5}j\right)/2 = \boxed{-\frac{1}{5}}\ \boxed{-\frac{3}{5}}j}$

P Q

$$S_{V_{g1}} = \frac{1}{2}(1)(\bar{I}_{m2}^{*}) = \frac{1}{2}(1)\left(-\frac{4}{5} - \frac{2}{5}j\right) = -\frac{4}{10} - \frac{2}{10}j =$$

CDU

$$= \underbrace{-\frac{2}{5}}_{P} \underbrace{-\frac{1}{5}}_{Q} j$$

$$S_{V_{g2}} = \frac{1}{2}(1+j)(\bar{I}_{m1}^{*}) = \frac{1}{2}(1+j)\left(-\frac{7}{10} - \frac{6}{10}j\right) =$$

$$= \frac{1}{20}(1+j)(-7-6j) = \frac{1}{20}(-7-6j-7j+6) =$$

CDU

$$= \underbrace{-\frac{1}{20}}_{P} \underbrace{-\frac{13}{20}}_{Q} j$$

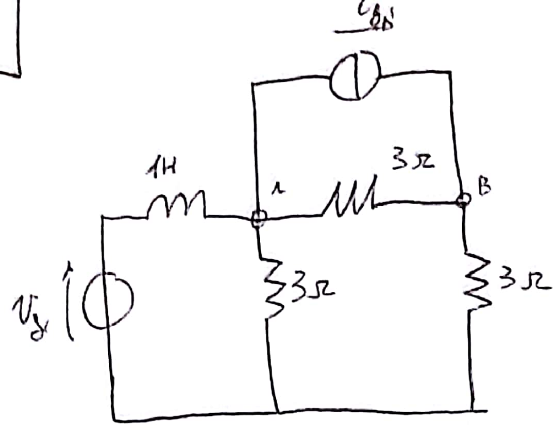
$$Q_L = \frac{1}{2} \cdot \mathrm{Im}\{Z_L\} |\bar{I}_L|^2 = \frac{1}{2}(2)\left(\frac{49}{100} + \frac{36}{100}\right) = \frac{85}{100} = \frac{17}{20}$$

CDU

$$Q_{V_{g1}} + Q_{V_{g2}} + Q_L = 0$$

$$\left(-\frac{1}{5}\right) + \left(-\frac{13}{20}\right) + \left(\frac{17}{20}\right) = 0$$

Es 5



$v_g(t) = \cos(2t) \rightarrow \bar{V}_g = 1$

$i_g(t) = \cos(2t) \rightarrow \bar{I}_g = 1$

$Y_L = -\frac{j}{\omega L} = -\frac{j}{2}$

Nodi

$$\begin{pmatrix} -\frac{j}{2}+\frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} \bar{V}_A \\ \bar{V}_B \end{pmatrix} = \begin{pmatrix} -\frac{j}{2}-1 \\ 1 \end{pmatrix}$$

$$\det: -\frac{j}{3} + \overbrace{\frac{4}{9}-\frac{1}{9}}^{3/9 = 1/3} = -\frac{j}{3}+\frac{1}{3}$$

V_B

$$\begin{vmatrix} -j/2 + 2/3 & -j/2 - 1 \\ -1/3 & 1 \end{vmatrix} = -j/2 + \frac{2}{3} - \frac{j}{6} - \frac{1}{3} = \frac{-3j+4-j-2}{6} = -\frac{4}{6}j + \frac{2}{6} = -\frac{2}{3}j + \frac{1}{3}$$

$$V_B = \frac{-2j+1}{\cancel{3}} \cdot \frac{\cancel{3}}{-j+1} = \frac{(-2j+1)(1+j)}{2} = \frac{-2j+2+1+j}{2} = \frac{3}{2} - \frac{j}{2}$$

$$P_R = \frac{1}{2} \operatorname{Re}\{Y_R\} \cdot |\bar{V}_B|^2 = \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{9}{4}+\frac{1}{4}\right) = \frac{10}{24} = \frac{5}{12} \text{ W}$$ ASS.